

Cascade Dual Frequency Smoothing for Local Area Augmentation System

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Abstract

Both temporal and spatial gradients of ionosphere errors have adverse effects upon local area augmentation system (LAAS). This article proposes a cascade dual frequency smoothing (CDFS) method to overcome this problem. One dual frequency smoothing filter is used to secure a precise estimate of ionosphere error, which is then used to modify ionosphere error in the code pseudorange measurement before it is put into another dual frequency smoothing (DFS) filter to attenuate noise. Thus, the ionosphere error is thoroughly removed from the smoothing process and excessive noise induced by GPS L2 code measurement is suppressed. Effectiveness of CDFS and accuracy of CDFS LAAS is analyzed based on data collected from LAAS test bed in Communication, Navigation, Surveillance/Air Traffic Management (CNS/ATM) Labs, Civil Aviation Administration of China (CAAC). The analysis demonstrates that CDFS method can eliminate residual errors in the smoothed pseudorange raised by the ionosphere temporal gradient and differential correction errors caused by the ionosphere spatial gradient.

Keywords: global positioning system; ionosphere; measurement errors; evaluation; estimation

1. Introduction

The local area augmentation system (LAAS), is a ground-based augmentation system to underpin the global positioning system (GPS), which uses the differential technique to compute a single correction for each satellite. The single correction includes all common errors between a local reference and users, and is the only way that has the high possibility of supporting Category III precise approach and landing^[1].

LAAS comprises ground and airborne subsystems^[2–3]. The ground subsystem of LAAS produces differential corrections for each satellite in view by combining individual measurements from each reference receiver and implements statistical monitor algorithms to detect and remove abnormality in special signals either in GPS or in ground subsystem itself. These corrections are transmitted to airborne subsystem and are used to modify the errors in the airborne pseudorange measurement, to enable elimination of

common errors in the ground and in the airborne subsystem and provide a more precise position solution.

As dominant terms in errors, receiver noise and multipath errors do not belong to the common errors shared by ground and airborne subsystems^[4], and they need to be removed from corrections. As such, a so-called smoothing technique is introduced into both ground and airborne subsystems to attenuate these errors^[5–6]. This process, in general, is low-pass filtering of the pseudorange measurements by means of changes in carrier phase to average out rapidly altering errors thereby leading to significant increase of accuracy of the smoothed filter output, called carrier smoothed code (CSC). The smoothing technique currently used for LAAS is also regarded as single frequency smoothing (SFS) since it utilizes GPS L1 code and carrier measurements only. Variation of ionosphere over time and space imposes two major limitations to SFS^[7–11]. First, the temporal gradient of ionosphere delay causes residual errors in CSC. Second, the spatial gradient of ionosphere delay induces additional errors in differential corrected pseudorange in airborne subsystem. These problems cannot be solved by SFS itself due to its single frequency nature.

To deal with the above-mentioned problems, P. Hwang and G. McGraw developed two dual frequency smoothing (DFS) methods on the basis of the dual frequency GPS data for smoothing, which are called

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divergence-free (DFree) and ionosphere-free (IFree) respectively^[12-13]. Either can remove adverse effects caused by ionosphere temporal gradient. Besides, IFree can also eliminate differential correction errors resulting from ionosphere spatial gradient but with higher noise level compared to DFree.

This article describes a cascade dual frequency smoothing (CDFS) technique. Simulation based on GPS observation data proves that, by removing the ionosphere error term in smoothing filter inputs, CDFS eliminates errors caused by ionosphere temporal and spatial gradient and keeps noise as low as DFree.

2. SFS and DFS

As background, this section gives a brief introduction of existing SFS and DFS techniques. Detailed description can be found in Refs.[12-14].

2.1. GPS observation model

GPS L1 code and the carrier phase measurement, ρ_1 and ϕ_1 respectively, are

$$\left. \begin{aligned} \rho_1 &= R + C + i_1 + n_{\rho_1} \\ \phi_1 &= R + C - i_1 + N_1 + n_{\phi_1} \end{aligned} \right\} \quad (1)$$

where R is the true geometric range between satellite and reference receiver antenna, C the sum of errors common to code and carrier phase measurement, including satellite clock error, ephemeris errors, and tropospheric errors, i_1 the L1 ionosphere refraction, N_1 the range ambiguity for the GPS L1 carrier, n_{ρ_1} the GPS L1 code and the carrier tracking noise, and n_{ϕ_1} the multipath.

Similarly, GPS L2 code and the carrier phase measurement, ρ_2 and ϕ_2 respectively, are

$$\left. \begin{aligned} \rho_2 &= R + C + i_2 + n_{\rho_2} \\ \phi_2 &= R + C - i_2 + N_2 + n_{\phi_2} \end{aligned} \right\} \quad (2)$$

The relationships between L1 and L2 ionosphere refractions can be accurately expressed by

$$\left. \begin{aligned} i_1 - i_2 &= \left(1 - \frac{f_{L1}^2}{f_{L2}^2} \right) i_1 = \alpha i_1 \\ i_2 - i_1 &= \left(1 - \frac{f_{L2}^2}{f_{L1}^2} \right) i_2 = \beta i_2 \end{aligned} \right\} \quad (3)$$

where f_{L1} and f_{L2} are GPS L1 and L2 carrier frequencies, respectively.

2.2. SFS

Fig.1 illustrates a carrier smoothing process^[12], where P and Φ represent the generalized code and carrier phase measurement, respectively, F the first-order filter characteristic of low-pass continuous-time

and fixed-gain with smoothing time constant of τ , Y the CSC output.

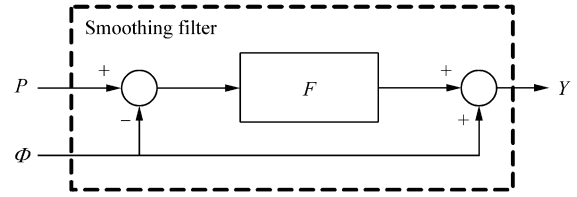


Fig.1 Carrier smoothing process.

SFS simply uses $P = \rho_1$ and $\Phi = \phi_1$ as inputs. Then, the output CSC will be

$$Y_s = R + C + I + n_s \quad (4)$$

where $I = (2F - 1)i_1$ is the ionosphere term and $n_s = Fn_{\rho_1} + (1 - F)n_{\phi_1}$ the noise.

The difference between the ionosphere terms in CSC and in raw code pseudorange is the ionospheric error term in CSC, which is defined as $\Delta I = I - i_1 = 2(F - 1)i_1$. As smoothing filter reaches its steady state, $\Delta I = 2\tau I_d$ is obtained, where I_d is the temporal gradient of ionospheric delay.

Assuming standard deviation of n_{ρ_1} is σ_{ρ_1} , the standard deviation of n_s is $\sigma_s^2 \approx \sigma_{\rho_1}^2 / (2N)$, where $N = \tau / T$, T is the interval between each individual measurement. Increasing τ would decrease n_s but increase ΔI . The smoothing time constant of LAAS, 100 s, is set to be a tradeoff between attenuation of code measurement errors and the effects of the ionosphere^[13].

2.3. SFS LAAS

Fig.2 illustrates LAAS differential process^[2-3,5]. The superscripts G and A are used to denote ground and airborne subsystem, respectively.

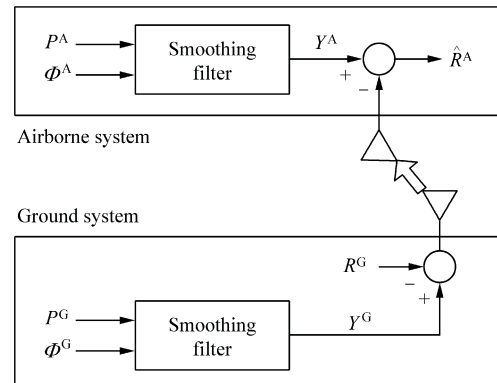


Fig.2 LAAS differential process.

Both ground and airborne subsystems use a smoothing filter to attenuate noise in code pseudorange measurements. The position of reference receivers in

ground subsystem has been precisely determined in advance. The ground subsystem can compute the position of each GPS satellite in real time using ephemeris, and, consequently, calculate R^G , the true geometry range from satellite to reference receiver. Based on the assumption that Y^A has approximately the same error with Y^G , the difference between Y^G and R^G , called differential correction, is treated as common error between ground and airborne subsystem. Thus, an estimation of true range between airborne subsystem and GPS satellite is given by

$$\hat{R}^A = Y^A - (Y^G - R^G) \quad (5)$$

Then, the differential correction error is

$$D = \hat{R}^A - R^A = Y^A - (Y^G - R^G) - R^A = (Y^A - R^A) - (Y^G - R^G) \quad (6)$$

Differential error of SFS LAAS, in which both ground and airborne systems use SFS for smoothing, can be written as

$$D_S = (C^A - C^G) + (I^A - I^G) + (n_S^A - n_S^G) \quad (7)$$

where $C^A - C^G$ is the difference of the sum of errors common to ground and airborne subsystem and approximates 0, $n_S^A - n_S^G$ the noise of the correction.

The ensuing ionosphere term deserves special attention:

$$I^A - I^G = 2(F^A i_1^A - F^G i_1^G) - (i_1^A - i_1^G) \quad (8)$$

LAAS requires that an identical smoothing filter be used in both ground and airborne subsystems^[15-18], that is, $F^A i_1^A \approx F^G i_1^G$. Consequently, the ionosphere term in differential correction is the ionosphere spatial gradient between ground and airborne system.

2.4. DFS

Two types of DFS, called DFree and IFree, were first suggested by P. Hwang and G. McGraw^[12-14].

Let inputs of DFree be

$$\left. \begin{aligned} P &= \rho_1 \\ \Phi &= \phi_1 - \frac{2}{\alpha}(\phi_1 - \phi_2) \end{aligned} \right\} \quad (9)$$

Then,

$$Y_D = R + C + i_1 + n_D \quad (10)$$

could be derived, where

$$n_D = F n_{\rho_1} + (1 - F) \left[n_{\phi_1} - \frac{2}{\alpha} (n_{\phi_1} - n_{\phi_2}) \right] \quad (11)$$

n_D is approximately the same as n_S , since they have the same code noise term. Besides, due to the fact that ionosphere error term in DFree CSC and raw code measurement are identical, the smoothing time constant can be greatly increased to reduce n_D without additional error associating with ionosphere temporal gradient.

The differential error of DFree LAAS is

$$D_D = (C^A - C^G) + (i_1^A - i_1^G) + (n_D^A - n_D^G) \quad (12)$$

The ionosphere spatial gradient between ground and airborne subsystems is still involved in D_D . But what is more important is that, application of DFree no longer requires an identical smoothing filter used in both ground and airborne subsystems. Therefore, the airborne subsystem can use measurements from one satellite before the smoothing filter reaches its steady state and the probability of loss of continuity could then be reduced.

Let inputs of IFree be

$$\left. \begin{aligned} P &= \rho_1 - \frac{1}{\alpha}(\rho_1 - \rho_2) \\ \Phi &= \phi_1 - \frac{1}{\alpha}(\phi_1 - \phi_2) \end{aligned} \right\} \quad (13)$$

Then IFree CSC could be expressed as

$$Y_I = R + C + n_I \quad (14)$$

where

$$n_I = F n_{\rho_1} + (1 - F) n_{\phi_1} \quad (15)$$

$$n_{\rho_1} = n_{\rho_1} - \frac{1}{\alpha}(n_{\rho_1} - n_{\rho_2}) \quad (16)$$

$$n_{\phi_1} = n_{\phi_1} - \frac{1}{\alpha}(n_{\phi_1} - n_{\phi_2}) \quad (17)$$

The standard deviation of n_I is

$$\sigma_I^2 \approx \frac{1}{2N} \left[\left(1 - \frac{1}{\alpha} \right)^2 \sigma_{\rho_1}^2 + \left(\frac{1}{\alpha} \right)^2 \sigma_{\rho_2}^2 \right] \quad (18)$$

Differential error of IFree LAAS is

$$D_I = (C^A - C^G) + (n_I^A - n_I^G) \quad (19)$$

Ionosphere error is thoroughly eliminated from IFree CSC and D_I contains no ionosphere spatial gradient error. However, standard deviation of n_I is about three times higher than n_S because GPS L2 code noise is introduced.

In conclusion, both types of DFS can eliminate adverse effects caused by ionosphere temporal gradient. Besides, IFree can eliminate differential errors caused by ionosphere spatial gradient but with larger noise.

By skillful designing, the smoothing filter could win advantages of both types of DFS. The next section discusses this in more detail.

3. CDFS

A user can estimate ionosphere error precisely by using measurements from multiple frequencies. Therefore, a multiple frequency LAAS user needs a differential correction containing no ionosphere error identical to the requirement of IFree. However, IFree uses $(\rho_1 - \rho_2)/\alpha$ as an approximation of i_1 to eliminate ionosphere error in filter inputs and therefore intro-

duces GPS L2 code noise. If i_1 can be precisely estimated and noise can be better treated, the performance of IFree could be further improved.

3.1. Estimate ionosphere error

For the purpose of improving differential positioning accuracy, wide area augmentation system (WAAS) is applied to acquire errors from different sources separately. To precisely estimate ionosphere error, WAAS uses a DFS filter, of which the inputs are

$$\left. \begin{aligned} P &= \frac{1}{\alpha}(\rho_1 - \rho_2) \\ \Phi &= -\frac{1}{\alpha}(\phi_1 - \phi_2) \end{aligned} \right\} \quad (20)$$

then

$$Y_0 = i_1 + n_0 \quad (21)$$

where

$$n_0 = \frac{1}{\alpha}F(n_{\rho_1} - n_{\rho_2}) + \frac{1}{\alpha}(F-1)(n_{\phi_1} - n_{\phi_2}) \quad (22)$$

The standard deviation of n_0 is

$$\sigma_0^2 = \frac{1}{2N} \left(\frac{1}{\alpha} \right)^2 (\sigma_{\rho_1}^2 + \sigma_{\rho_2}^2) \quad (23)$$

Noise of Y_0 is only about $\sqrt{1/(2N)}$ of $(\rho_1 - \rho_2)/\alpha$, therefore making it a better candidate as an estimation of i_1 than $(\rho_1 - \rho_2)/\alpha$.

3.2. CDFS

Benefit can be gained if Y_0 is used to eliminate ionosphere errors and use smoothing filter to attenuate noise in raw code measurements. Since output of the smoothing filter to estimate ionospheric error becomes part of inputs of another filter, this technique is called CDFS.

Inputs of CDFS are

$$\left. \begin{aligned} P &= \rho_1 - Y_0 \\ \Phi &= \phi_1 - \frac{1}{\alpha}(\phi_1 - \phi_2) \end{aligned} \right\} \quad (24)$$

then

$$Y_C = R + C + n_C \quad (25)$$

where

$$n_C = Fn_{\rho_1} + Fn_0 + (1-F)n_{\phi_1} \quad (26)$$

The standard deviation of n_C is

$$\begin{aligned} \sigma_C^2 &\approx \frac{1}{2N}(\sigma_{\rho_1}^2 + \sigma_0^2) = \frac{1}{2N} \left[\sigma_{\rho_1}^2 + \frac{1}{2N} \left(\frac{1}{\alpha} \right)^2 \cdot \right. \\ &\quad \left. (\sigma_{\rho_1}^2 + \sigma_{\rho_2}^2) \right] = \frac{1}{2N} \sigma_{\rho_1}^2 + \left(\frac{1}{2N} \right)^2 \cdot \\ &\quad \left(\frac{1}{\alpha} \right)^2 (\sigma_{\rho_1}^2 + \sigma_{\rho_2}^2) \end{aligned} \quad (27)$$

Similar to IFree, there is no longer an error associating with ionosphere in CDFS CSC. CDFS will not suffer from error related to ionosphere temporal gradient. n_C contains two parts. The first part is approximate to n_D . The second part is a high order infinitesimal of the first part. That is, as N increases, the second part will reduce more rapidly than the first part. Consequently, performance of CDFS will improve greatly if N is longer.

Differential error of CDFS LAAS is

$$D_C = (C^A - C^G) + (n_C^A - n_C^G) \quad (28)$$

Error caused by ionosphere is eliminated.

Thus, CDFS combines advantages of both DFree and IFree, the advantages being:

(1) Elimination of residual smoothing error caused by ionosphere temporal gradient by removing ionosphere error from filter inputs.

(2) Elimination of differential error caused by ionosphere spatial gradient by removing ionosphere term from CSC.

(3) Reduction of noise in CSC by cascade filtering of code noise.

4. Performance Analysis

Simulation is done to evaluate performance of smoothing techniques discussed earlier based on a total of 300 GPS data set, each containing 10 000 continuous code and carrier measurements, collected from the LAAS test bed developed in Communication, Navigation, Surveillance/Air Traffic Management Labs, Civil Aviation Administration of China (CAAC).

4.1. Smoothing performance analysis

In simulation, an ionosphere temporal gradient of 1 mm/s^[19] is injected from the 5 000th to 10 000th samples in each set.

Fig.3 shows residual errors of SFS in presence of ionosphere temporal gradient with different smoothing time constant. There are obvious deviations after ionosphere temporal gradient is injected. As smoothing time constant increases, noise in CSC reduces but the deviation increases and yields to $2\tau I_d$, as discussed in Section 2.2.

Fig.4 illustrates different behaviors of SFS, DFS, and CDFS in presence of ionosphere temporal gradient. Smoothing time constant is set to 400 to all filters. Neither DFS nor CDFS comprises deviation in their residual errors, proving that DFS and CDFS can remove adverse effects caused by ionosphere temporal gradient. Noise of both IFree and CDFS are higher than that of DFree.

Fig.5 shows residual errors of ionosphere error estimation dependent upon different smoothing time constants. When compared with standard deviation of

error of $(\rho_1 - \rho_2)/\alpha$ as an estimation of i_1 , it is 4.078 m. The result closely matches the value predicted in Section 3.1.

Fig.6 shows residual errors of DFree, IFree, and CDFS dependent upon different smoothing time constants.

From Fig.6, it follows that IFree has the highest noise, whereas DFree has the least. When the smoothing time constant is 100 s, noise of IFree is about 2.4 times that of DFree. DFree and IFree decrease the noise at almost equal rates as the smoothing time constant increases. When smoothing time constant increases from 100 s to 1 000 s, the noise of DFree and IFree drops by a factor of 79.83%.

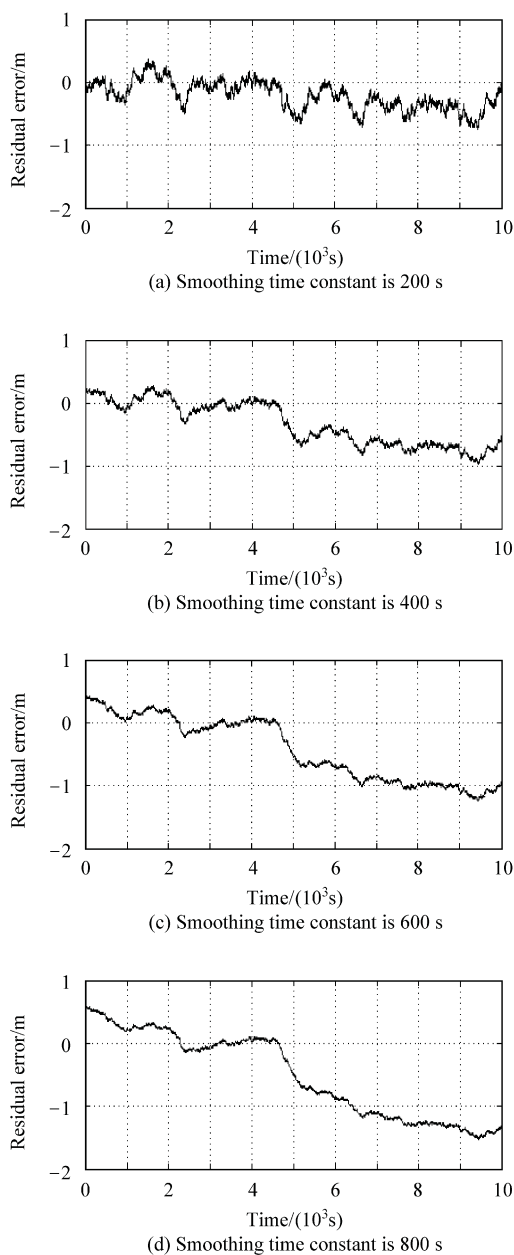


Fig.3 Effects of ionosphere temporal gradient on single frequency smoothing.

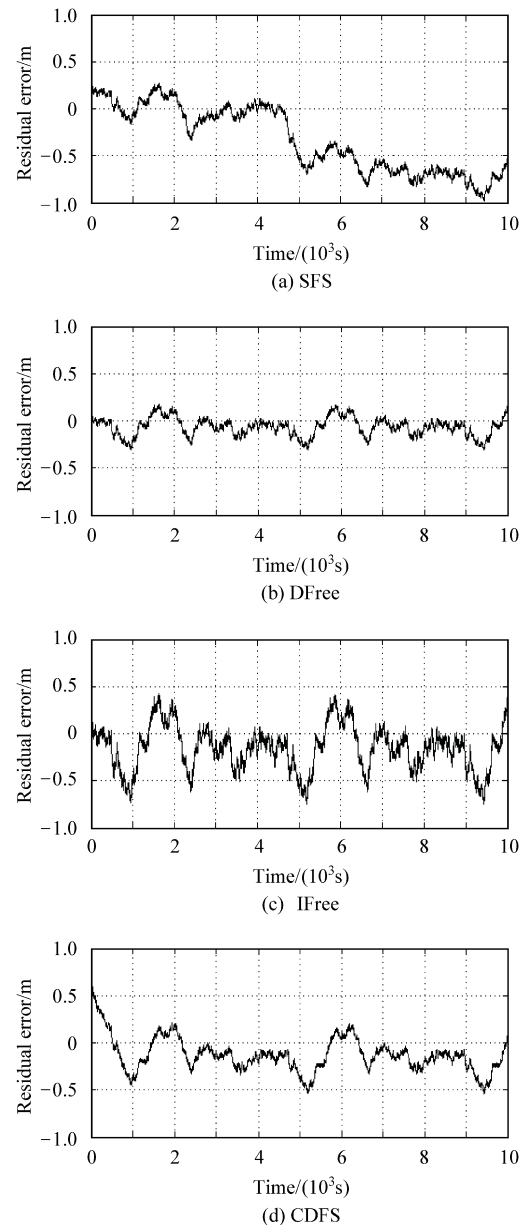


Fig.4 Effects of ionosphere temporal gradient on SFS, DFS, and CDFS.

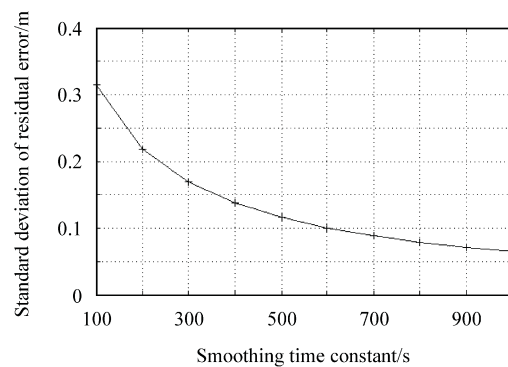


Fig.5 Residual errors of ionosphere error estimation.

Noise of CDFS is about 77.25% of IFree and 1.85 times of DFree when smoothing time constant is 100 s.

However, as the smoothing time constant further increases, the noise of CDFS would reduce faster than DFree and IFree. When the smoothing time constant reaches 1 000 s, the noise of CDFS is only 55.54% of IFree and 1.38 times of DFree.

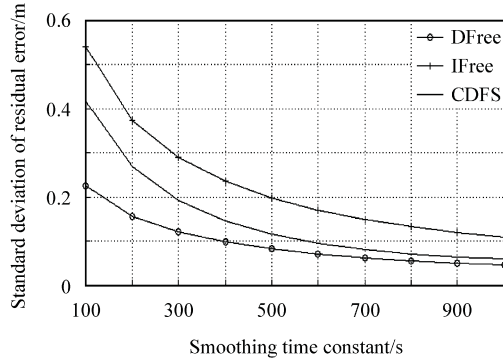


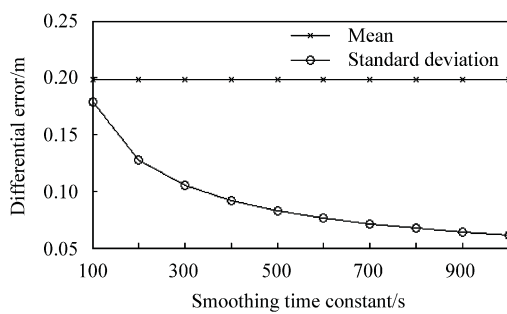
Fig.6 Residual errors of DFree, IFree, and CDFS dependent upon different smoothing time constants.

4.2. Differential performance analysis

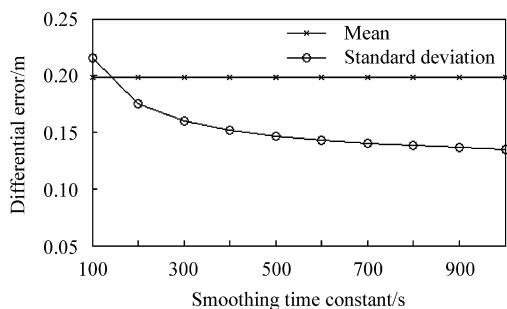
It is assumed that the ionosphere spatial gradient between ground and airborne subsystems in simulation is 0.01 m/km and the distance between them is 20 km.

Fig.7 shows differential error of LAAS based on SFS, DFree, IFree, and CDFS with different smoothing time constants, respectively.

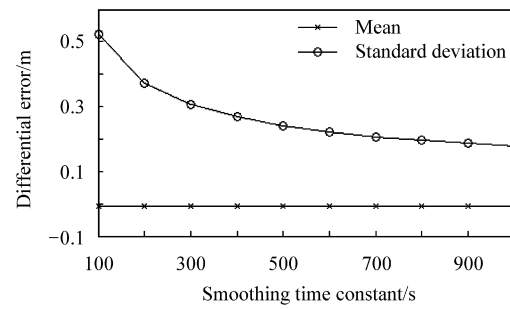
Mean of SFS LAAS differential error is 0.2 m, which is equivalent to the error caused by ionosphere spatial gradient between ground and airborne subsystems. The standard deviation, which indicates noise level reduces, as the smoothing time constant increases, but still remains at least 0.2 m.



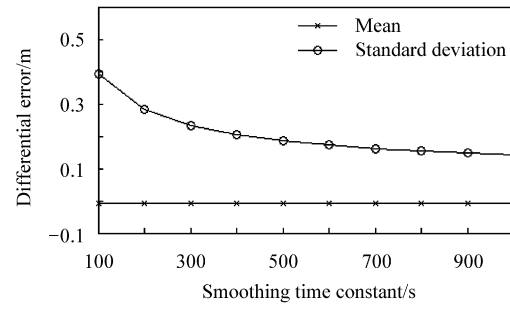
(a) SFS



(b) DFree



(c) IFree



(d) CDFS

Fig.7 Differential error of LAAS.

Similar to SFS, in DFree LAAS differential errors, there exist errors caused by ionosphere spatial gradient between ground and airborne subsystems.

Note that the mean values of IFree LAAS differential errors stay unchanged at zero all the time, which implies that IFree has eliminated differential errors caused by ionosphere spatial gradient, but the flip side is an obvious increase of noise.

Similar to IFree LAAS, CDFS LAAS eliminates differential errors caused by ionosphere spatial gradient, and yet its noise is much less than that of IFree LAAS. When the smoothing time constant increases to 1 000 s, the noise of CDFS LAAS is equivalent to that of DFree LAAS.

5. Conclusions

This article has introduced CDFS technique. CDFS reduces CSC noise by way of precise estimation of ionosphere error derived from WAAS smoothing filter.

The simulation bears witness to the ensuing conclusions:

- (1) CDFS eliminates residual smoothing errors caused by ionosphere temporal gradient by removing ionosphere error from filter inputs.
- (2) CDFS eliminates differential errors caused by ionosphere spatial gradient by removing ionosphere term from CSC.
- (3) CDFS reduces noise in CSC by cascade filtering of GPS L2 code noise.

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